The relationship between image noise and spatial resolution of CT scanners

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Image noise and spatial resolution

- Aims
  - Describe the origins of the ImPACT Q factor
  - Explore the proportionality relationships
    - in particular; noise against resolution
  - From the findings, look at some implications

The ImPACT Q factor

- Describes a relationship of image quality with respect to dose

$$Q \; \alpha \; \sqrt{\frac{f^3}{\sigma^2 z \; D}}$$

f = spatial resolution

σ = image noise

z = slice width

D = dose

- High Q factor
  - good 'image quality' with low dose
  - high spatial resolution, low noise, narrow slice

The ImPACT Q factor

- Drawn from the proportional relationship

$$\sigma^2 \; \alpha \; \frac{f^3}{z \, D}$$

f = spatial resolution expressed as a frequency (c/cm)

σ = image noise

z = slice width

D = dose

Theoretical derivation

- Rodney Brookes and Giovanni di-Chiro (1976)

  Statistical limitations in x-ray reconstructive tomography

  Medical Physics Vol 3, No 4 July 1976

- Riederer S.J., Pelc N.J. and Chesler D.A. (1978)

  The Noise Power Spectrum in Computed Tomography

  Physics in Medicine and Biology 1978 23(3), 446-454

$$\sigma^2(\mu) = \frac{\pi^2 \ \beta \ \gamma(E) \ e^{\alpha} \ \mu_{en} \ E}{1200 \ \omega^3 z \ D}$$

Other publications, reports and books

- Bassano D.A. (1980, AAPM Summer School)
  - Specification and Quality Assurance for CT Scanners
- IPEM TGR 32 iii (1st Edition, 1981)
  - Measurement of Performance Characteristics of CT Scanners
- Farr RF and Allisy-Roberts PJ (1997)
  - Physics for Medical Imaging
- Seeram E., 2000
  - Computed Tomography, Physical Principles, Clinical Applications and Quality Control (2nd Edition)

$$\sigma^2 \;\alpha\; \frac{f^3}{z\, D}$$

Unpacking the relationship

- noise relationship with number of photons
  - established

$$\sigma^2 \; \alpha \; \frac{1}{N}$$

- number of photons
  - proportional to slice thickness and mAs (dose)

$$\sigma^2 \; \alpha \; \frac{1}{z \; D}$$

Unpacking the relationship

- noise relationship with spatial resolution ?

$$\sigma^2 \; \alpha \; f^3 \qquad \text{...?}$$

$$\boxed{\sigma^2 \; \alpha \; \frac{f^3}{z\,D}}$$

Empirical approach

- measure image noise and spatial resolution
- range of convolution kernels

@ constant dose and slice width

$$\sigma^2 \; \alpha \; \frac{f^3}{z \; D}$$

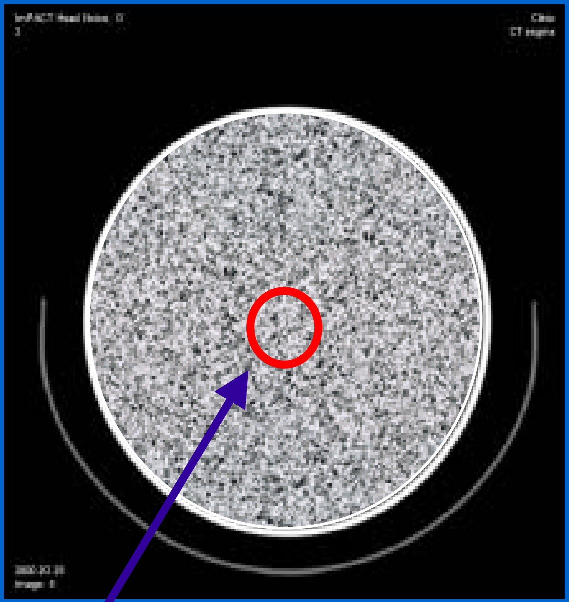
Image noise

water filled phantom

scan projection radiograph

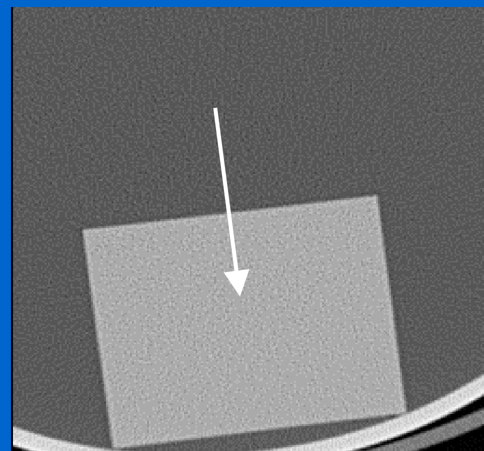
scan

image



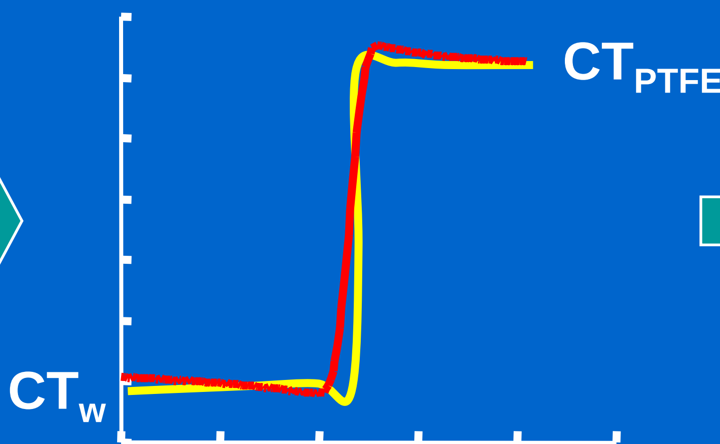
'noise' = standard deviation (σ) of CT values in region of interest

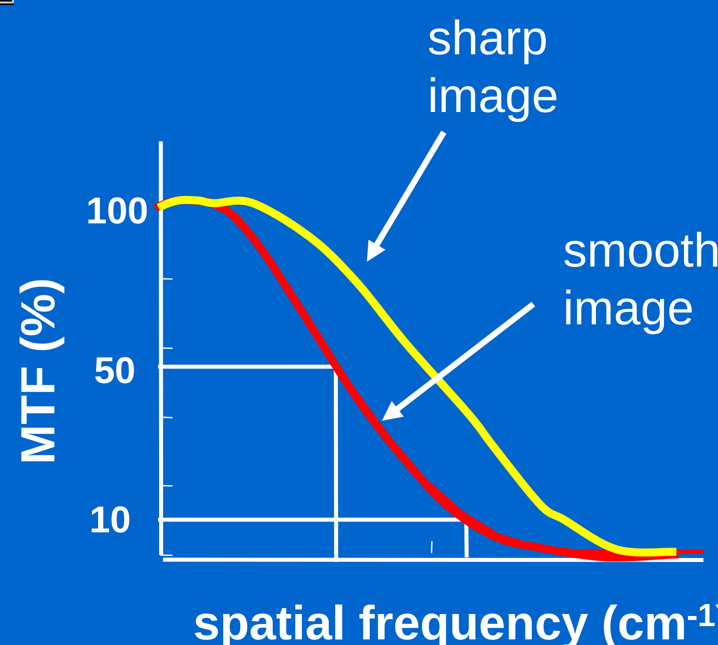
Spatial resolution

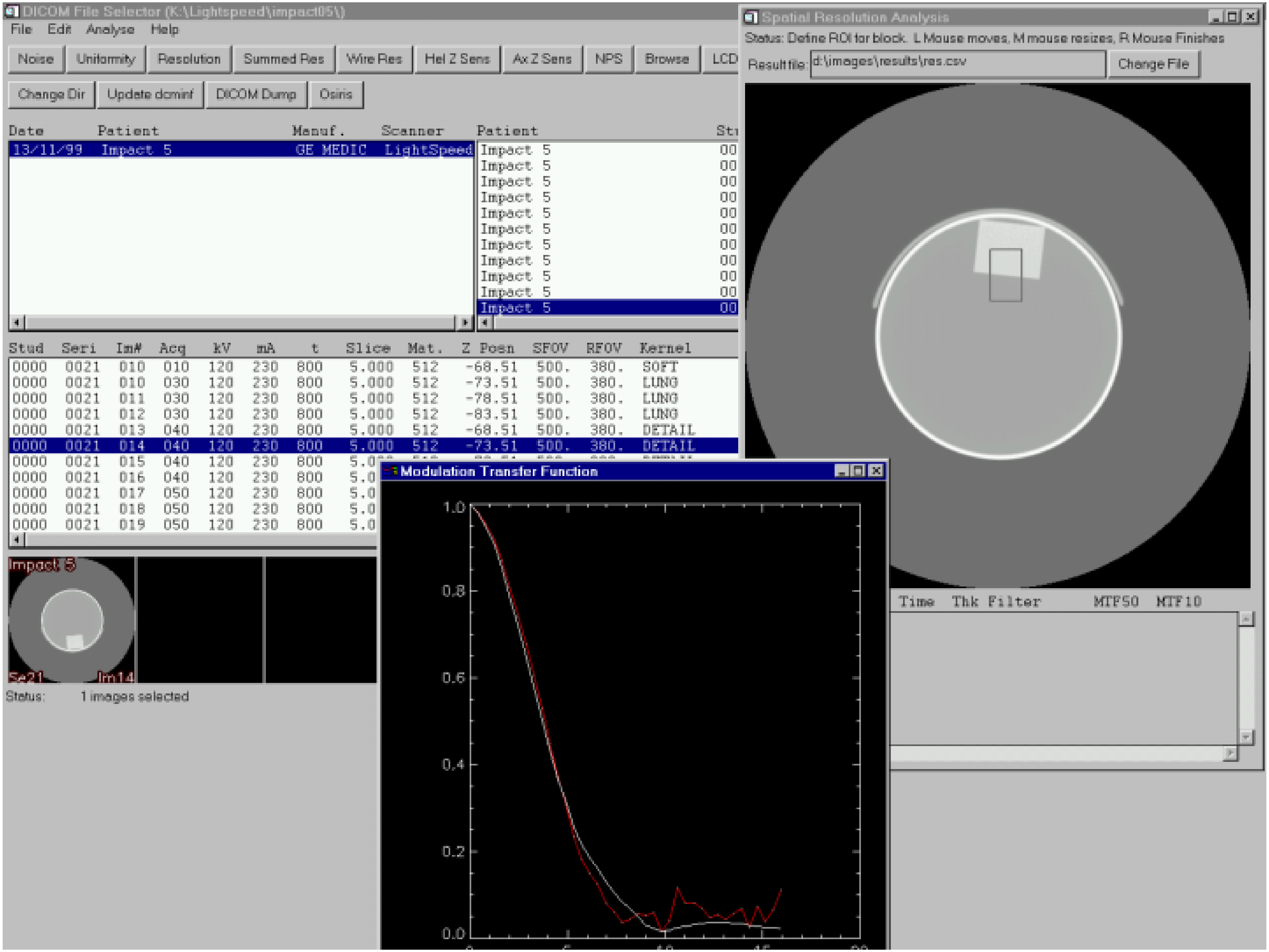


high contrast edge

ESF $\rightarrow$ MTF







DICOM File Selector (K:\Lightspeed\impact05\)
File Edit Analyse Help
Noise Uniformity Resolution Summed Res Wire Res Hel Z Sens Ax Z Sens NPS Browse LCD
Change Dir Update dcminf DICOM Dump Osiris
Date Patient Manuf. Scanner Patient
13/11/99 Impact 5 GE MEDIC LightSpeed
Stud Seri Im# Acq kV mA t Slice Mat. Z Posn SFOV RFOV Kernel
0000 0021 010 010 120 230 800 5.000 512 -68.51 500. 380. SOFT
0000 0021 010 030 120 230 800 5.000 512 -73.51 500. 380. LUNG
0000 0021 011 030 120 230 800 5.000 512 -78.51 500. 380. LUNG
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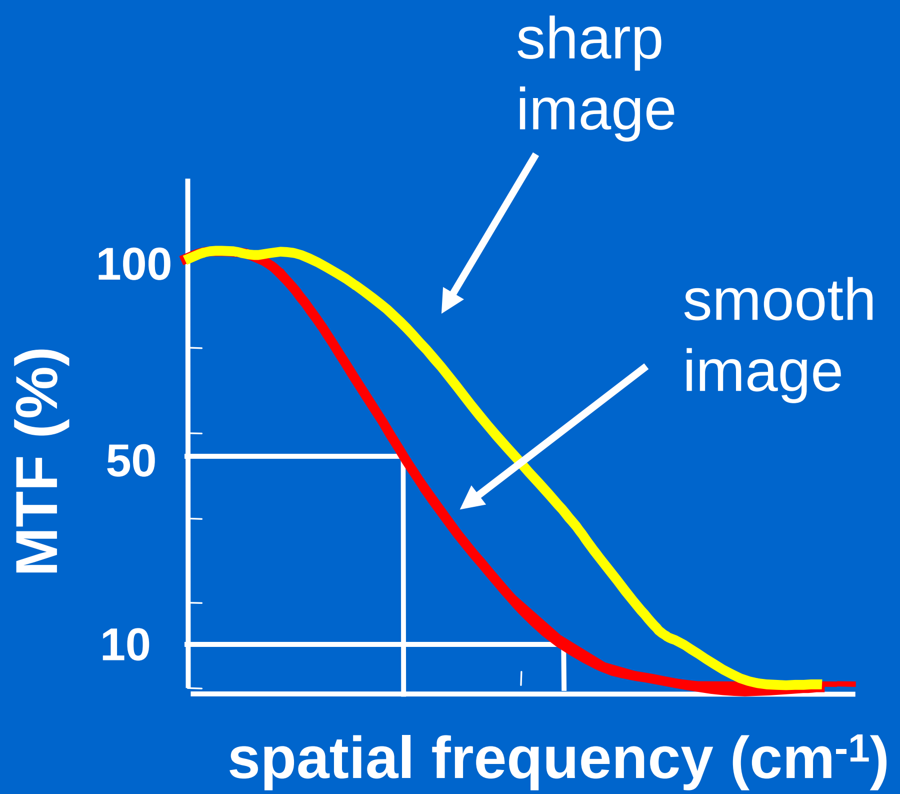
Impact 5
Se21 Im14
Status: 1 images selected
Spatial Resolution Analysis
Status: Define ROI for block. L Mouse moves, M mouse resizes, R Mouse Finishes
Result file: d:\images\results\res.csv
Change File
Time Thk Filter MTF50 MTF10
Modulation Transfer Function
1.0
0.8
0.6
0.4
0.2
0.0

Modulation transfer function

- resolution descriptors
  - frequency at MTF_{50}
  - frequency at MTF_{10}
  - Q uses average of MTF_{50}, MTF_{10}

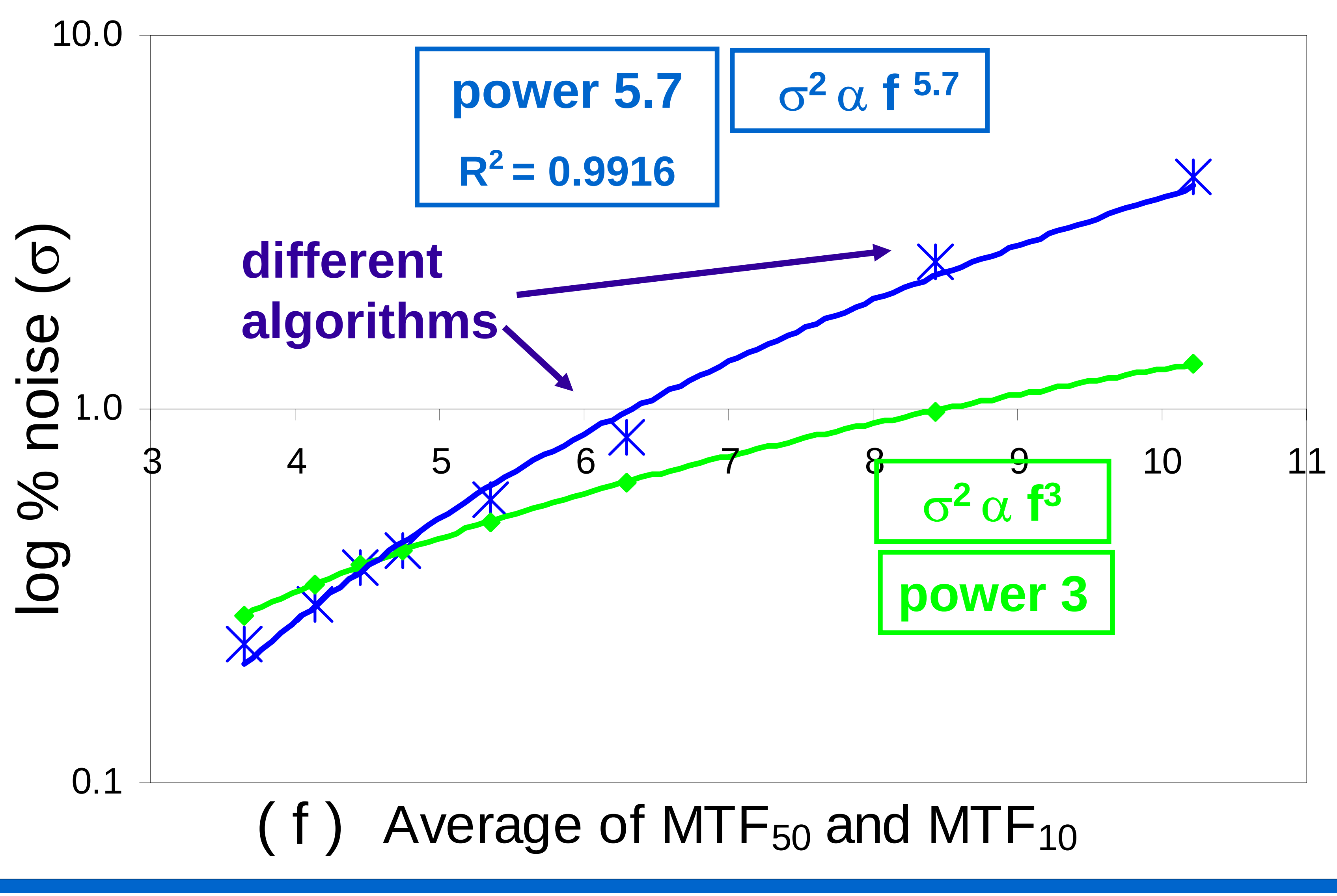


- more recently
  - MTF_{80}
  - MTF_2 ≈ 'cut-off'
  - integral (area under curve)

Scanners and algorithms used

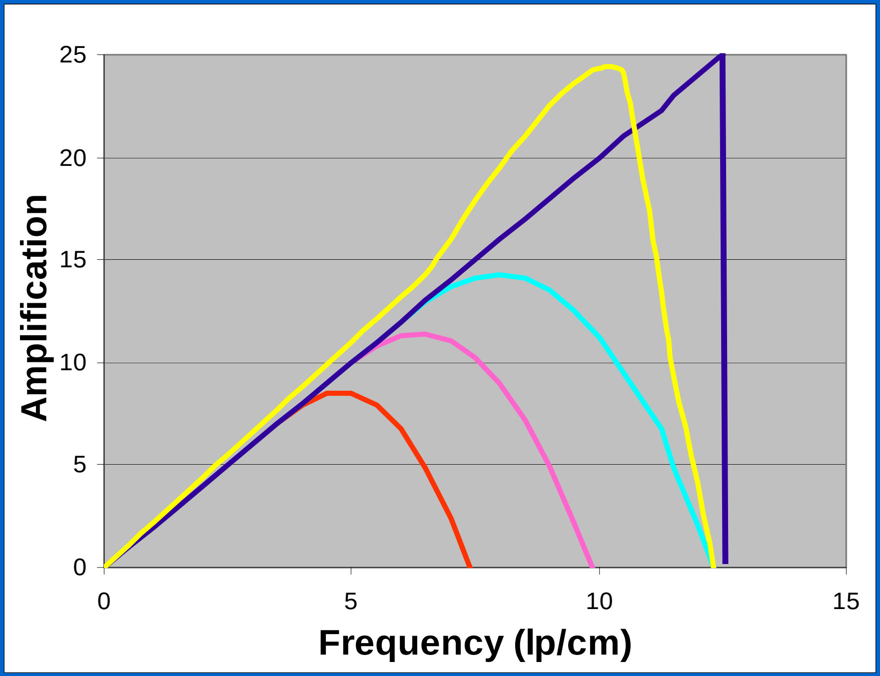
- IGE Lightspeed
  - h: soft, standard, lung, detail, bone, edge
  - b: soft, standard, lung, detail, bone, edge
- Siemens Volume Zoom
  - h: AH..10,20,30,40,50,60,70
  - b: AB...10,20,30,40,50,60
- Toshiba Aquilion
  - h: FC…20,21,22,23,24,25,26,27,28,30,80
  - b: FC….10,11,12,13,14,30
- Marconi (Philips) MX8000
  - h: A,EB,EC,B,C,D
  - b: A,EC,B,C,D

Siemens VZ, all algorithms, head scans

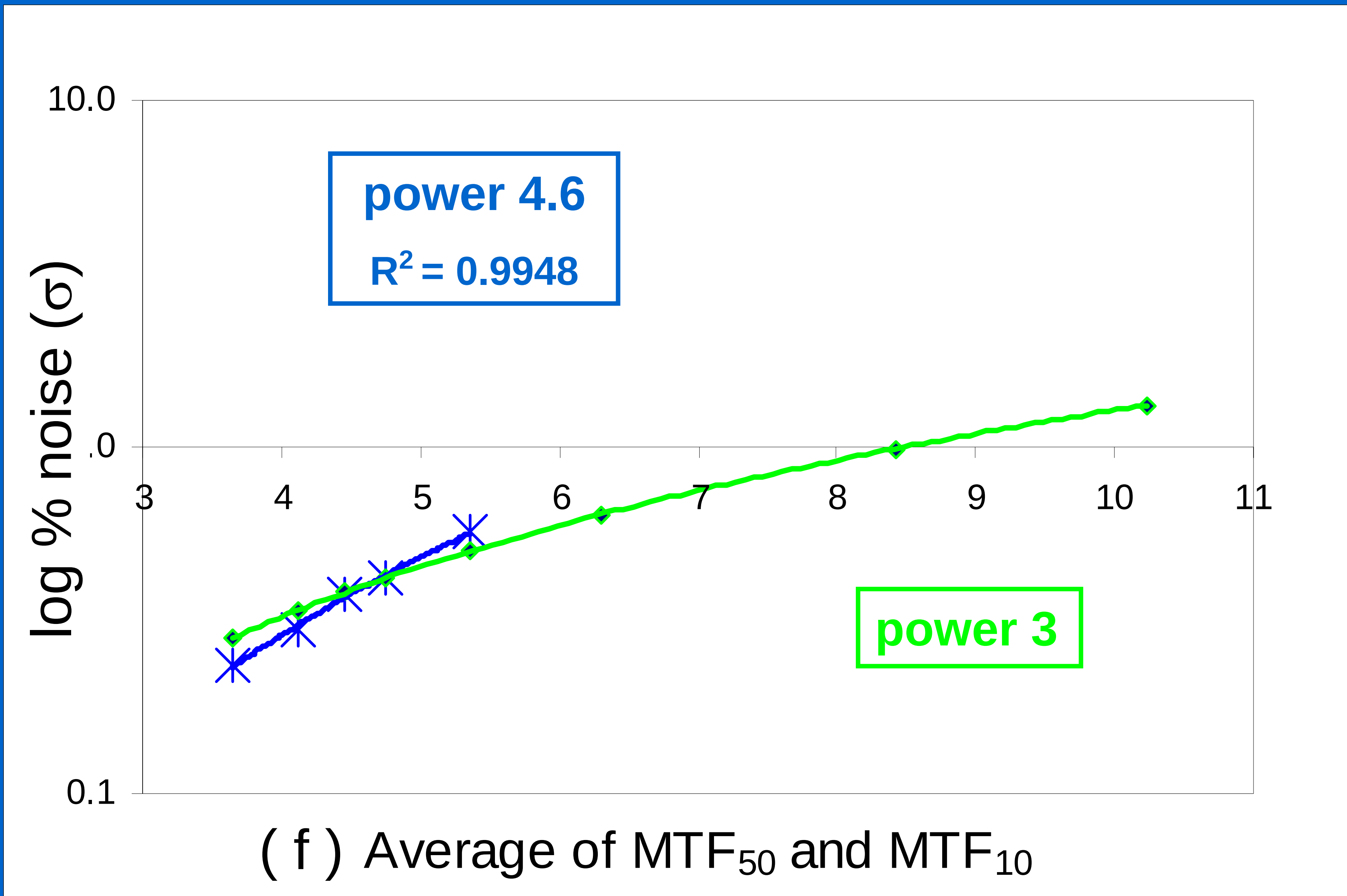


Empirical view of noise versus resolution

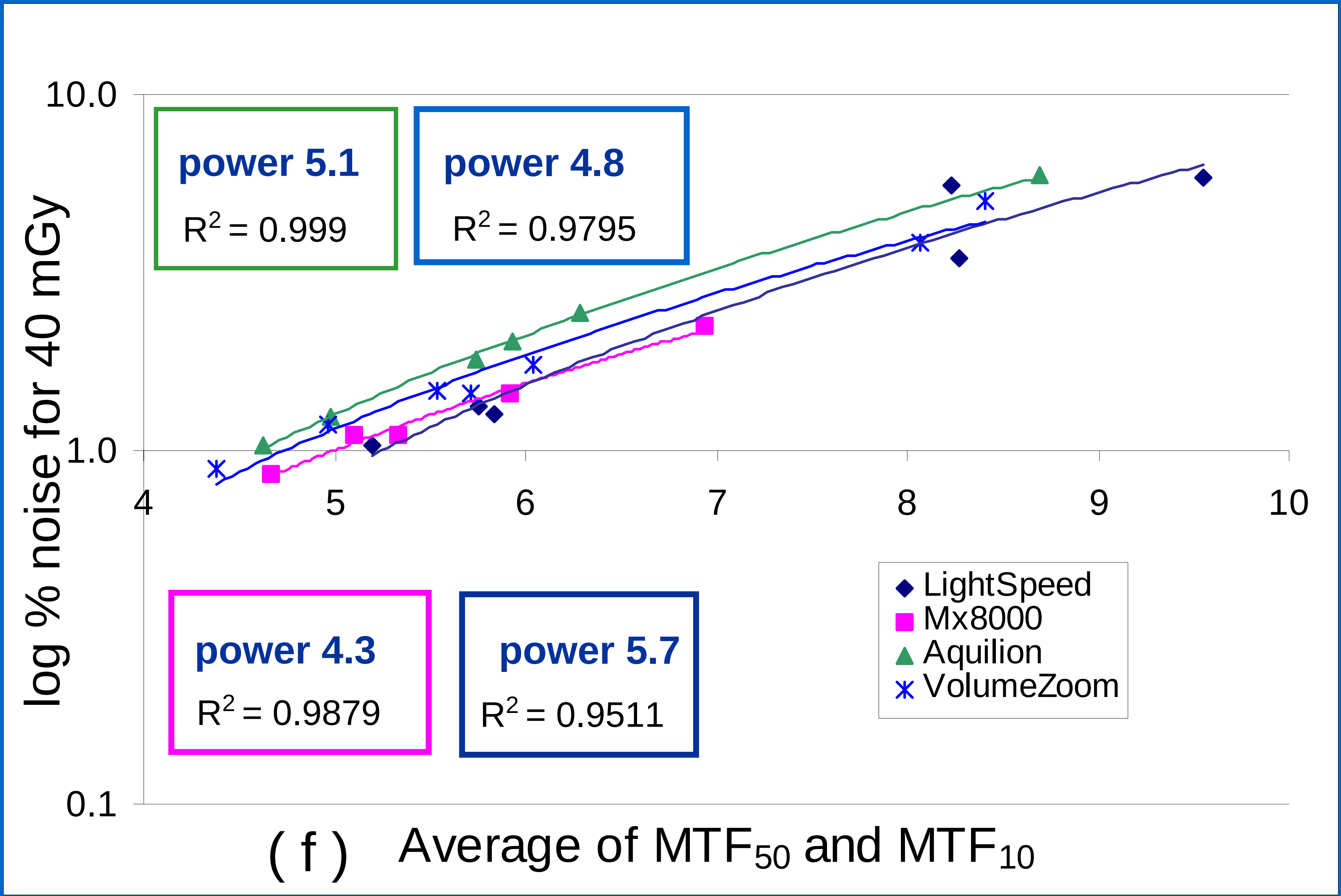
- early papers only considered simple algorithms
  - ramp filter and Hanning weighted
  - some filter frequencies boosted very differently



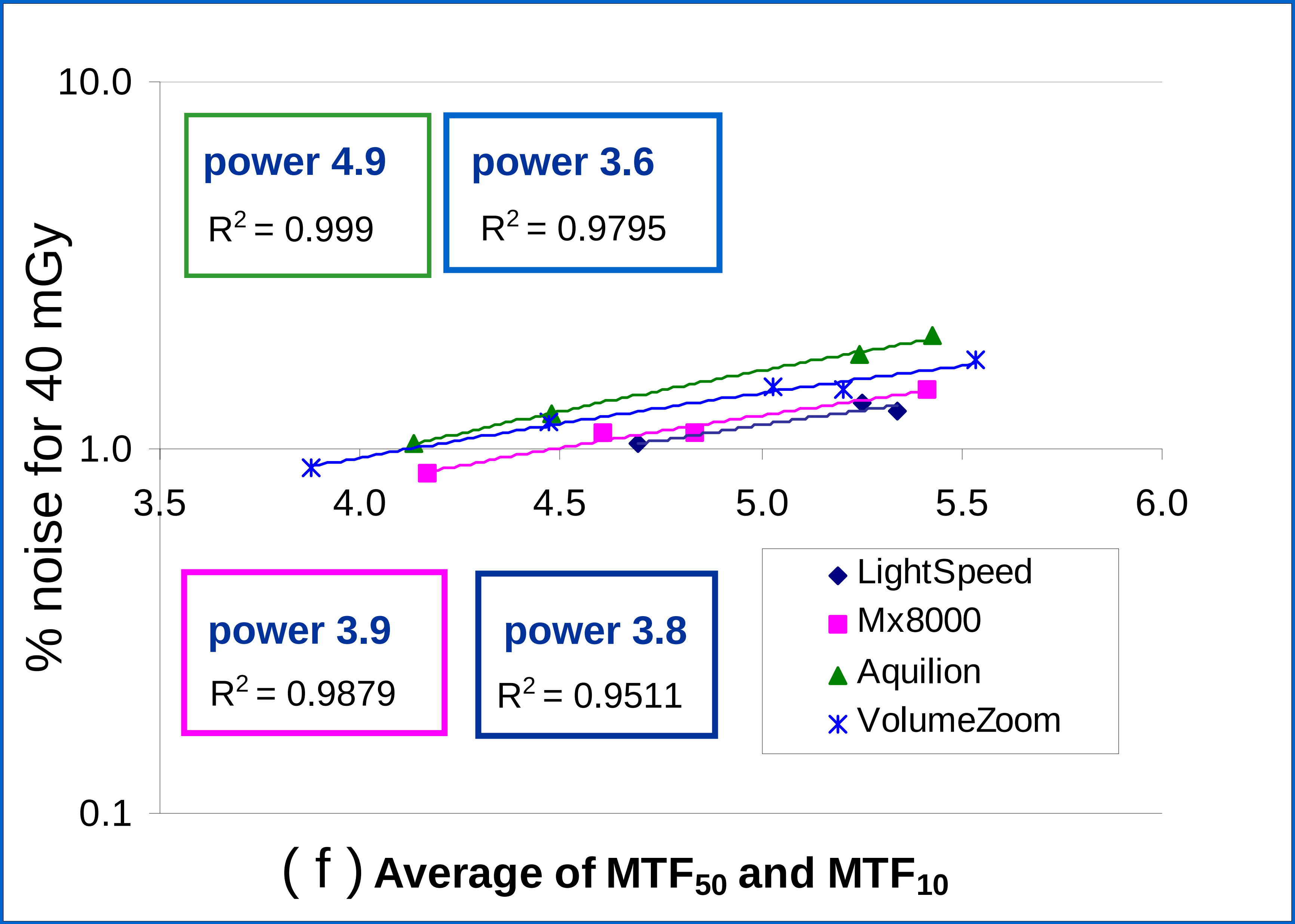
Siemens VZ, low res. algorithms, head scans



All scanners, all algorithms, body scans

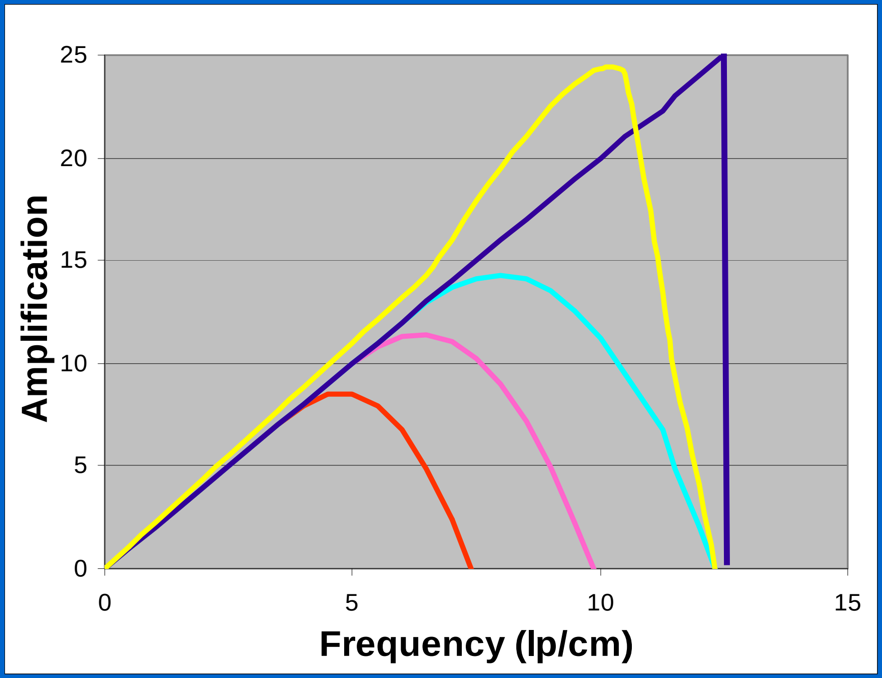


All scanners, low res. algs, body scans



Empirical view of noise versus resolution

- early papers
  - derivations focussed on limiting resolution characteristics eg detector aperture



Mean of all scanners, all MTF, all algorithms

Body scans

parameter	resolution power for sigma^2	
	mean	*range +/-*
mtf_{80}	3.0	*0.7*
mtf_{50}	4.2	*0.2*
mtf_{10}	5.2	*1.1*
mtf_{2}	3.0	*1.6*
$mtf_{integral}$	3.9	*0.4*
$avg_{MTF50,10}$	5.0	*0.7*

Empirical view of noise versus resolution

- power factor for MTF_{50} and MTF_{10} is greater than 3; ~ between 4 and 5

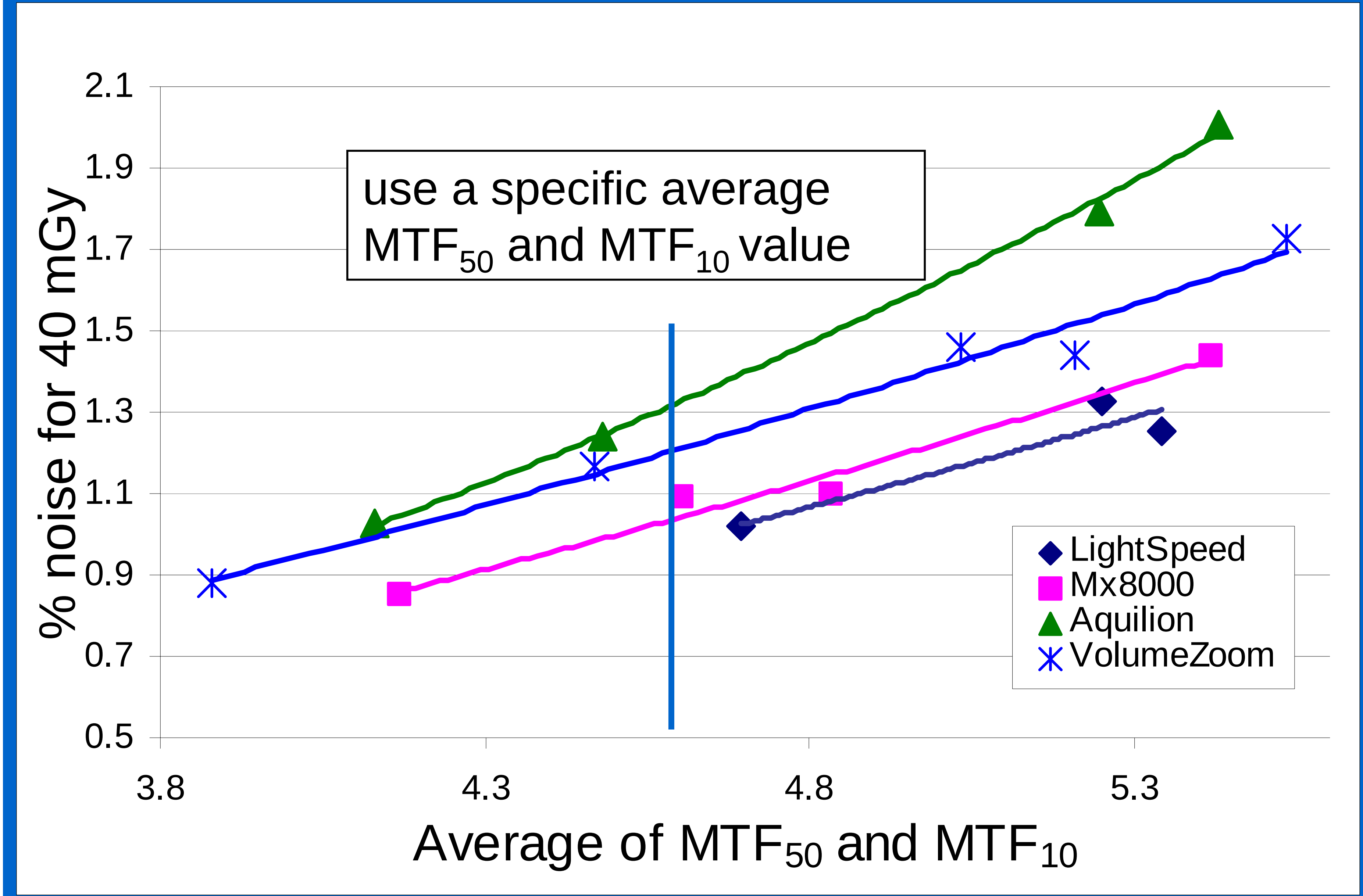
$$\sigma^2 \; \alpha \; \frac{f^{\approx 4-5}}{z \; D}$$

Empirical view of noise versus resolution

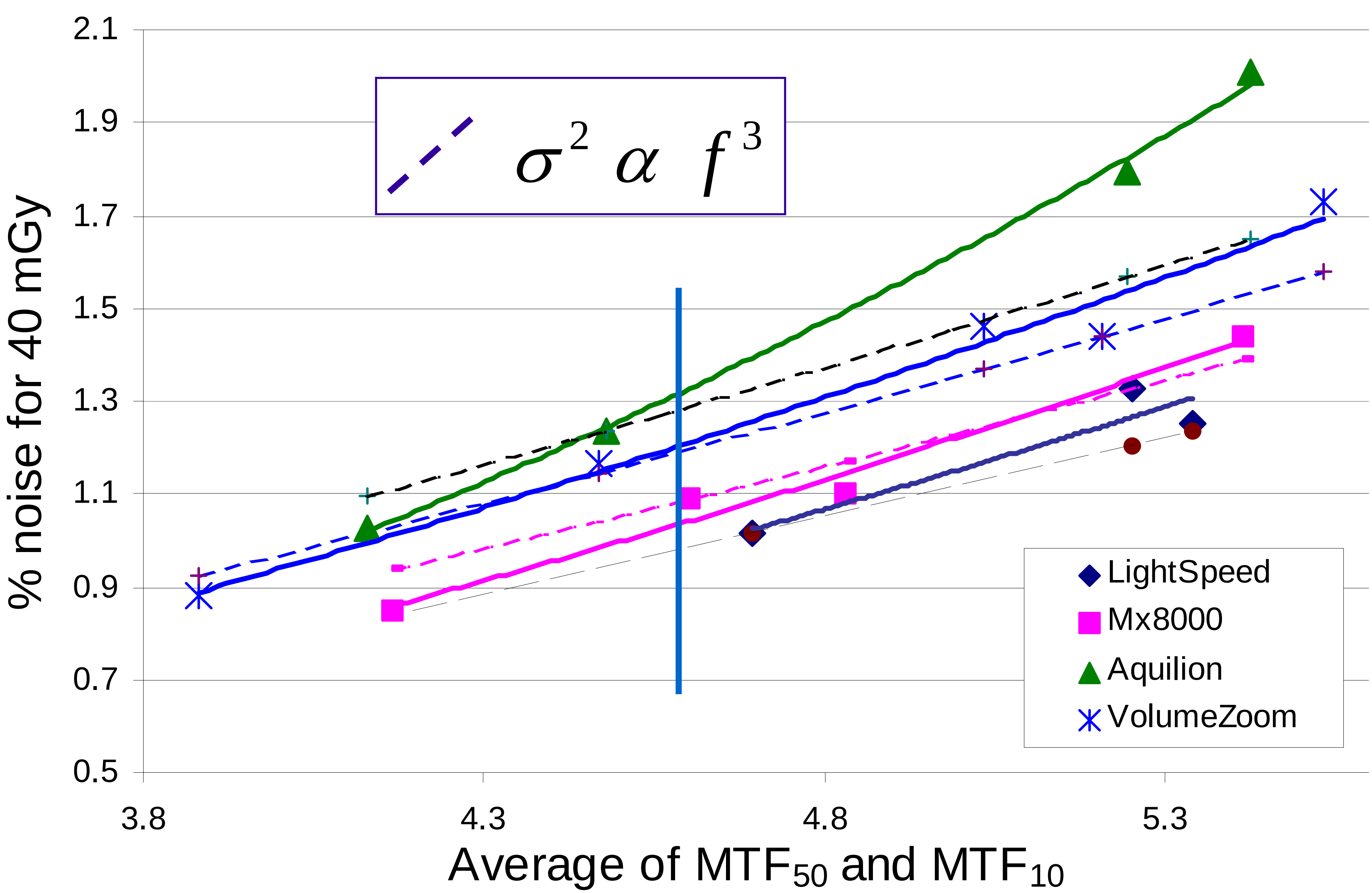
- how does this affect the calculation of Q ?
- minimise the influence of the power relationship
  - specify resolution
    - eg. body resolution values of MTF_{50} = 3.4 c/cm, MTF_{10} = 6 c/cm
  - find algorithm giving resolution data closest to that value
  - calculate Q

$$Q \; \alpha \; \sqrt{\frac{f^3}{\sigma^2 z \; D}}$$

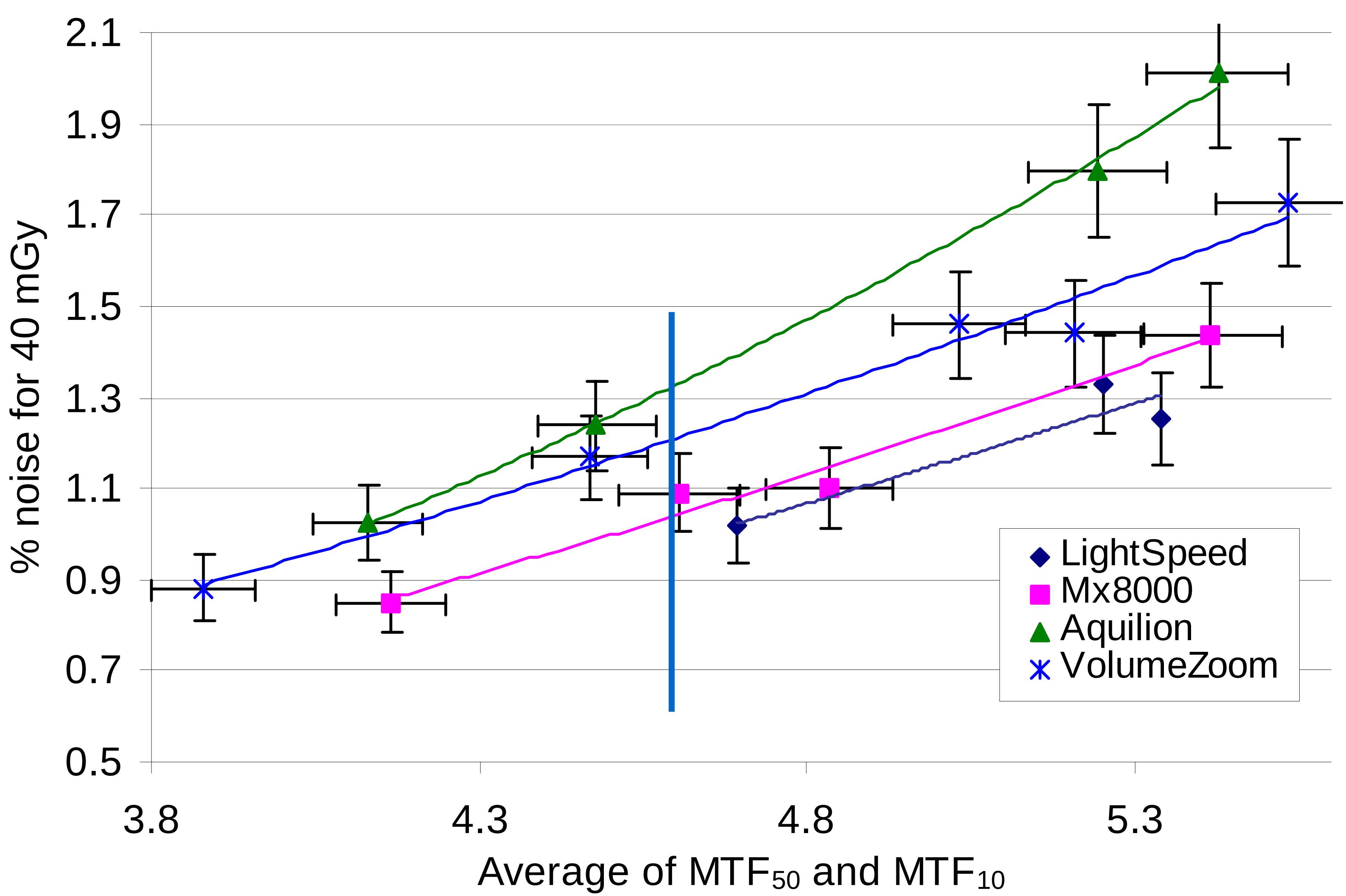
All scanners, body scans, lower resolution algorithms



All scanners, body scans, lower resolution algorithms



All scanners, body scans, lower resolution algorithms



Conclusion

- The ImPACT Q factor relies on an established relationship

$$\sigma^2 \; \alpha \; \frac{f^3}{z \; D}$$

- using average resolution parameters from the MTF, the noise squared relationship to resolution is shown to be to a power greater than 3

$$\sigma^2 \; \alpha \; \frac{f^{\approx 4-5}}{z \; D}$$

- by choosing algorithms close to a fixed spatial resolution the algorithm dependence is minimised

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Theoretical Relationships

- Rodney Brookes and Giovanni di-Chiro

energy absorption co-efficient

Logarithmic attenuation

Statistical error in
reconstructed image
i.e. image noise

average depth
dose factor

photon
energy

$$\sigma^2(\mu) = \frac{\pi^2 \ \beta \ \gamma(E) \ e^{\alpha} \ \mu_{en} \ E}{1200 \ \omega^3 h \ D}$$

beam spreading factor
(non parallel rays)

dose

detector aperture

slice width

Theoretical Relationships

$$\sigma^2(\mu) = \frac{\pi^2 \; \beta \; \gamma(E) \; e^{\alpha} \; \mu_{en} \; E}{1200 \; \omega^3 z \; D}$$

Microtomography - Basics

$$Time \propto \frac{SNR^2}{\rho\mu r^4}\exp\left(\frac{\mu d}{2}\right)$$

SNR = signal-to-noise resolution
ρ = density
μ = linear attenuation coefficient
r = voxel dimension
d = object diameter

Courtesy of Dr. E.J. Morton, Department of Physics, University of Surrey

Mean of all scanners, average $MTF_{50,10}$

	all algorithms		low res. agorithms	
	mean	*range +/-*	mean	*range +/-*
body	5.0	*0.7*	4.1	*0.6*
head	5.5	*0.3*	4.4	*1.2*

- $MTF_{50,10}$ lower by 20% with lower res. algs.
- Head ~ 10% higher than body
- Power factor between 4 and 5

The ImPACT Q factor

- Describes image quality with respect to dose
- High Q (quality) factor
  - good 'image quality' at low dose
  - image quality in terms of
    - high spatial resolution, low noise, narrow slice

$$Q \ \alpha \ \sqrt{\frac{f^3}{\sigma^2 z \ D}}$$

f = spatial resolution

σ = image noise

z = slice width

D = dose

Unpacking the relationship

- $$\sigma^2 \;\alpha\; \frac{f^3}{z\ D}$$

- noise relationship with number of photons
  – established

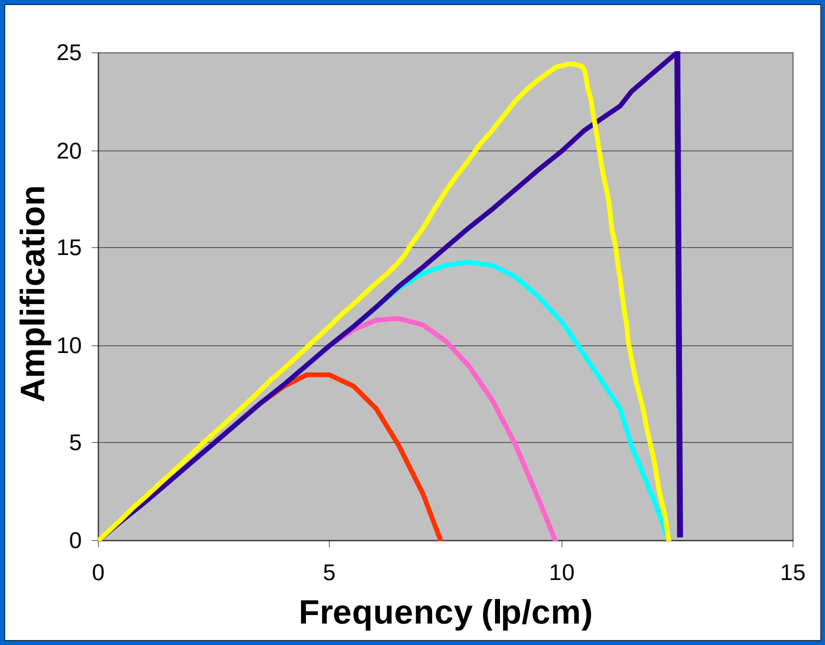
$$\sigma^2 \;\alpha\; \frac{1}{N}$$

- number of photons
  – proportional to slice thickness and mAs (dose)

$$\sigma^2 \;\alpha\; \frac{1}{z\ D}$$

Empirical view of noise versus resolution

- power factor is greater than 3; ~ between 4 and 5
- early papers only considered simple algorithms
  - ramp filter and Hanning weighted
  - some filter frequencies boosted very differently
  - theory looked at limiting resolution



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  Physics in Medicine and Biology 1978 23(3), 446-454

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